SEMESTER SYSTEM COURSE STRUCTURE
FOR
M.SC. IN MATHEMATICS
(PURE AND APPLIED STREAMS)

Under Choice Based Credit System (CBCS)
Effective from the academic session 2023-2025

DEPARTMENT OF MATHEMATICS
KANYASHREE UNIVERSITY
KRISHNAGAR, NADIA, PIN-741101

Course Structure

Semester	Type of Course	Paper Name	Credit	Marks	Total
I	CC-101 Theory	Real Analysis	4	50	Marks= 300 Credits =24
	CC-102 Theory	Complex Analysis	4	50	
	CC-103 Theory	Linear Algebra	4	50	
	CC-104 Theory	Ordinary Differential Equations and Special Functions	4	50	
	CC-105 Theory	Classical Mechanics	4	50	
	CC-106 Theory	Numerical Analysis	4	50	
II	CC-207 Theory	Abstract Algebra	4	50	Marks= 300 Credits =24
	CC-208 Theory	Functional Analysis	4	50	
	CC-209 Theory	Topology	4	50	
	CC-210 Theory	Partial Differential Equations	4	50	
	CC-211 Theory	Operations Research	4	50	
	SEC Practical	Computer-aided Numerical practical Using C	4	50	
III	CC-312 Theory	Graph Theory, Integral Equation and Integral Transformation	4	50	Marks= 300 Credits =24
	CC-313 Theory	Continuum Mechanics	4	50	
	CC-314 Theory	Calculus of $\mathbb{R}^n$	4	50	
	CC-315 Theory	Differential Manifold (P)	4	50	
		Dynamical Systems (A)			

	CC-316 Theory	Operator Theory (P)	4	50	
		Theory of Electromagnetic Fields (A)			
	GEC	Statistical Techniques	4	50	
IV	DSE-417	Elective Course-1	4	50	Marks= 300 Credits =24
	DSE-418	Elective Course-2	4	50	
	DSE-419	Elective Course-3	4	50	
	DSE-420	Elective Course-4	4	50	
	Project	Project Work • Dissertation • Seminar Presentation • Viva-voce	8	 50 30 20	

***P=Pure Stream, A=Applied Stream**

❖ The DSE Courses (DSE-417 to DSE-420) in fourth semester be offered to the students of both streams (Pure and Applied) on the basis of availability of Teachers and within the Framed Syllabi of the Optional Courses.

Discipline Specific Elective Courses

Course	Pure Stream	Applied Stream
DSE-417	Advanced Functional Analysis-I Advanced Differential Geometry-I	Advanced Operations Research-I Mathematical Biology-I
DSE-418	Advanced Complex Analysis-I Advanced Algebra-I	Fluid Mechanics-I Theory of Relativity
DSE-419	Advanced Functional Analysis-II Advanced Differential Geometry-II	Advanced Operations Research-II Mathematical Biology-II
DSE-420	Advanced Complex Analysis-II Advanced Algebra-II	Fluid Mechanics-II Quantum Mechanics

Detailed Syllabus:

Semester-I

Core Course-101: Real Analysis

Cardinal number: Definition, Schröder-Berstein theorem, Order relation of cardinal numbers, Arithmetic of cardinal numbers, Continuum hypothesis. (4)
Cantor's set: Construction and its presentation as an uncountable set of measure zero. (2)
Functions of bounded variation: Definition and basic properties, Lipschitz condition, Jordan decomposition, Nature of points of discontinuity, Nature of points of non-differentiability. (4)
Absolutely continuous functions: Definition and basic properties, Deduction of the class of all absolutely continuous functions as a proper subclass of all functions of bounded variation, Characterization of an absolutely continuous function in terms of its derivative vanishing almost everywhere. (4)
Riemann-Stieltjes integral: Basic properties and conditions for existence, integration by parts, change of variable, reduction to a Riemann integral, step function as integrator, reduction to a finite sum, mean value theorems, Helly's Second theorem on convergence of integral with respect to integrator. (6)
The Lebesgue measure: Definition of the Lebesgue outer measure on the power set of R, countable subadditivity, Carathéodory's definition of the Lebesgue measure and basic properties. Measurability of an interval (finite or infinite), Countable additivity, Characterizations of measurable sets by open sets, $G_\delta$sets, closed sets and $F_\sigma$sets. Measurability of Borel sets. (10)
Measurable functions: Definition on a measurable set in R and basic properties, Simple functions, Sequences of measurable functions, Measurable functions as the limits of sequences of simple functions, Lusin's theorem on restricted continuity of measurable functions, Egoroff's theorem, Convergence in measure. (5)
The Lebesgue integral: Integrals of non-negative simple functions, The integral of non-negative measurable functions on arbitrary measurable sets in R using integrals of non-negative simple functions, Monotone convergence theorem, The integral of measurable functions and basic properties, Absolute character of the integral, Dominated convergence theorem. (5)

References:
1. W. Rudin: Principles of Mathematical Analysis
2. H. L. Royden: Real Analysis.
3. B. K. Lahiri and K. C. Ray: Real Analysis.
4. W. Sierpinsky: Cardinal Number and Ordinal Number.
5. I. P. Natanson: Theory of Integrals of a Real Variable (Vol. I and II).
6. Malik and Arora: Mathematical Analysis

Core Course-102: Complex Analysis

Riemann's Riemann's sphere, stereographic projection, point at infinity and the extended complex plane. (2)
Functions of a complex variable, limit and continuity. Analytic functions, Cauchy-Riemann equations. Complex integration. Cauchy's fundamental theorem and its consequences. Cauchy's integral formula. (4)

Derivative of an analytic function. Morera's theorem. Cauchy's inequality. Liouville's theorem, Fundamental theorem of classical algebra. (6)
Uniformly convergent series of analytic functions. Power series. Taylor's theorem. Laurent's theorem. (8)
Zeros of an analytic function. Singularities and their classification. Limit points of zeros and poles. Riemann's theorem. Weierstrass-Casorati theorem. Theory of residues. Argument principle. Rouche's theorem. Maximum modulus theorem. Schwarz lemma. Open mapping theorem. Behaviour of a function at the point at infinity. (10)
Contour integration. Conformal mapping, Mobius transformation. Idea of analytic continuation. (6)
Multivalued functions – branch point. Idea of winding number. (4)

References:

1. A. I. Markushevich **:** Theory of Functions of a Complex Variable (Vol. I, II and III).
2. R. V. Churchill and J. W. Brown **:** Complex Variables and Applications.
3. E. C. Titchmarsh **:** The Theory of Functions.
4. E. T. Copson **:** An Introduction to the Theory of Functions of a Complex Variable.
5. J. B. Conway **:** Functions of One Complex Variable.
6. L. V. Ahlfors **:** Complex Analysis.
7. H. S. Kasana **:** Complex Variables – Theory and Applications.
8. S. Narayan and P. K. Mittal **:** Theory of Functions of a Complex Variable.
9. A. K. Mukhopadhyay **:** Functions of Complex Variables and Conformal Transformation.
10. J. M. Howie **:** Complex Analysis.
11. S. Ponnusamy, Foundation of Complex Analysis.
12. H. A Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
13. E. M. Stein and R. Shakrachi, Complex Analysis, Princeton University Press.

Core Course-103: Linear Algebra

Inner product spaces: Definition and examples of inner product spaces, Cauchy-Schwartz and triangle inequalities, norm, Orthogonality, Construction of orthogonal sets - The Gram-Schmidt process, Orthogonal complements, best approximation. [8H]

Linear transformations: Definition and examples of linear transformations, subspaces, rank-nullity theorem, Algebra of linear transformations, Isomorphism, Matrix representation of linear transformations, change of basis, similarity, solution of a system of equations, linear functional, dual space, Annihilator, Double dual, Transpose of a linear transformation. [12H]
Canonical form of similarity: Condition for diagonalization, eigenvalues and eigenvectors, eigen spaces, similar and congruent matrices, characteristic polynomial, minimal polynomial, annihilating polynomials, diagonalization, diagonalization of Hermitian matrices, reduction of a matrix to normal form, Jordan Canonical form. [10H]
Bilinear and Quadratic forms: Definitions and examples, matrix associated with a bilinear form, quadratic form, reduction to the canonical form, rank, signature and index of a quadratic form, reduction of a quadratic form to normal form, Sylvester's law of inertia, simultaneous reduction of two quadratic forms, applications to analytical geometry-geometrical interpretation of eigenvalues and eigenvectors of real symmetric matrix, Conic sections in R2, Quadric surfaces. [10H]
Complexifications: Complexification of a vector space, complexification of an operator, minimal polynomial of the complexification, eigenvalues of the complexification, characteristic polynomial of the complexification, normal operators on real inner product spaces, isometries

on the real inner product spaces, characterization of normal operators over real field, trace of an operator. [10H]

References:

1. S. H. Friedberg, A. J. Insel and L. E. Spence, Linear Algebra (4th Edition), Prentice-Hall of India Private Limited, New Delhi (2012).
2. K. Hoffman and R. Kunze, Linear Algebra (Second Edition), Prentice-Hall of India Private Limited, New Delhi (1996).
3. Sheldon Axler, Linear Algebra done right, Springer, 3rd edition (2015).
4. S. Kumaresan, Linear Algebra: A Geometrical Approach, Prentice-Hall of India Private Limited, New Delhi (2000).
5. A. R. Rao and P. Bhimasankaram, Linear Algebra, Hindustan Book Agency, New Delhi, (2000).
6. S. K. Berberian, Linear Algebra. Oxford University Press, (1992).
7. S. Lang, Linear Algebra., Springer, 3rd edition, (1987).

Core Course-104: Ordinary Differential Equations and Special Functions

Solution of linear differential equations of nth order. Wronskian, linear dependence and independence of the solution set. Fundamental set of solutions. Abel's identity. Method of variation of parameters. Normal form. (7)
Initial value problem. Boundary value problem. Picard's method of successive approximation. The Lipschitz condition. Picard's existence and uniqueness theorem for solution of initial value problem. Gronwall's lemma and its applications. Fundamental 14
Inequality. Green's function for boundary value problem and solution of non-homogeneous linear equations. (8)
Adjoint and self-adjoint equations. Lagrange's identity. Regular Sturm-Liouville problems for second order linear equations. Eigen values and eigen functions. Expansion in eigen functions. Sturm's separation theorem. Sturm's comparison theorem. (5)
Solution of second order linear differential equations in complex domain. Ordinary points and singular points. Fuch's theorem. Frobeneous Theorem (Statement only). Introduction to special functions. (5)
Legendre Polynomial: Generating function, Recurrence relations, Rodrigue's formula, Orthogonal property. Schlafli's integral formula. Laplace's first and second integral formula. Construction and solution of Legendre differential equation. (3)
Bessel's function: Generating function, Recurrence relation, Representation for the indices ½, -1/2, 3/2 and -3/2. Bessel's integral equation. Bessel's function of second kind. Solution of Bessel's differential equation (3)
Hermite Polynomial: Generating function, Recurrence relations, Rodrigue's formula, Orthogonal property. Construction and solution of Hermite differential equation. (3)
Laguerre Polynomial: Generating function, Recurrence relations, Rodrigue's formula, Orthogonal property. Construction and solution of Laguerre differential equation. (3)
Chebyshev Polynomial: Definition, Series representation, Recurrence relations, Orthogonal property. Construction and solution of Chebyshev differential equation. (3)

References :

1. G. F. Simmons : Differential Equations.
2. E. E. Coddington and N. Levinson : Theory of Ordinary Differential Equations.
3. M. Birkhoff and G. C. Rota **:** Ordinary Differential Equations.
4. M.D. Raisinghania : Advanced Differential Equations.

5. E. L. Ince **:** Ordinary Differential Equations
6. N. N. Lebedev **:** Special Functions and Their Applications.
7. I. N. Sneddon **:** Special Functions of Mathematical Physics and Chemistry.
8. E. D. Rainville **:** Special Function

Core Course-105: Classical Mechanics

Rotating frames of reference: Rotating coordinate system, motion of a particle relative to rotating earth, Coriolis force, deviation of freely falling body from vertical, Foucault's pendulum. [4]
Motion of a rigid body: Two-dimensional motion of a rigid body rotating about a fixed point - velocity, angular momentum and kinetic energy, Euler's dynamical equations and its solutions, invariable line and invariable plane, torque free motion, Euler's angles,
Components of angular velocity in terms of Euler's angles, motion of a top in a perfectly rough floor, stability of top motion. [6H]
Constrained motion: Constraints and their classification with examples, Lagrange's equation of motion of the first kind, Gibbs-Appell's principle of least action, D'Alembert's principle. [4H]
Lagrangian mechanics: Degrees of freedom, generalised coordinates, Lagrange's equations of motion of the second kind (holonomic and non-holonomic systems), velocity dependent potential, dissipative forces, Rayleigh's dissipation function, generalised momenta and energy, gauge function for Lagrangian, invariance of the Euler-Lagrange equations (under coordinate transformation, Galilean transformation), cyclic coordinates, Routh process for ignorable coordinates, symmetry and conservation laws. [8H]
Hamiltonian mechanics: Legendre dual transformation, Hamilton's canonical equations of motion, properties of Hamilton's function, principle of least action, Hamilton's principle, derivation of the Euler-Lagrange's equations of motion, derivation of the Hamilton's equations of motion, invariance of Hamilton's principle under coordinate transformation. [8H]
Calculus of Variations: Derivation of Euler-Lagrange's equation, sufficient condition for existence of extremals, Brachistochrone problem, geodesic, isoperimetric problem, variational problems with moving boundaries. [8H]
Canonical transformations: Definition, examples and properties of canonical transformations, generating functions, Liouville's theorem, Poisson bracket (definition and properties), Poisson's theorems, Condition of canonicality in terms of Poisson bracket, Lagrangian bracket, Poisson's bracket of angular momentum, Infinitesimal canonical transformations, Hamilton-Jacobi theory, Hamilton's principle and characteristic functions, Noether's theorem. [8H]
Small oscillations: Small oscillations in systems with more than one degree of freedom, Normal coordinates, Oscillations under constraints, Oscillations with dissipation, Forced oscillations. [4]

References:

1. H. Goldstein, Classical Mechanics, Narosa Publ. House, 1997.
2. V. B. Bhatia, Classical Mechanics with introduction to nonlinear oscillation and chaos, Narosa Publishing House, 1997.
3. N. C. Rana & P.S. Jog, Classical Mechanics, Tata McGraw Hill, 2001.
4. A. S. Gupta, Calculus of Variations with Applications, Prentice –Hall of India, 1996.
5. E. T. Whittaker – A Treatise on the Analytical Dynamics of Particles and Rigid Bodies, Cambridge University Press, 1993.
6. D. T. Green Wood – Classical Dynamics, Dover Publication, 2006.
7. F. R. Gantmakher – Lectures in Analytical Mechanics, Mir Publishers, 1970
8. J. L. Synge & B. a. Graffith, Principles of Mechanics, Mc. Graw-Hill Book Co. 1960.
9. I. M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall Inc, 2012.

Core Course-106: Numerical Analysis

Errors: Floating-point approximation of a number, Loss of significance and error propagation, Stability in numerical computation.(4)
Interpolation: Hermite's and spline interpolation. Interpolation by iteration – Aitken's and Neville's schemes. (4)
Polynomial Approximation: Least square approximation. Weighted least square approximation.
Orthogonal polynomials, Gram – Schmidt orthogonalisation process, Chebysev polynomials, Mini-max polynomial approximation.(5)
Numerical Integration: Gaussian quadrature formula and its existence. Euler- MacLaurin formula, Gregory-Newton quadrature formula, Richardson extrapolation, Romberg integration, Improper integrals. (5)
Systems of Linear Algebraic Equations: Direct methods, Factorization method, Successive-Over Relaxation (SOR) iteration method, Convergence, Concept of ill conditioned systems. (4)
Eigen value and Eigenvector Problems: Direct methods, Iterative method–Power method, Inverse Power method, Jacobi method and convergences. (3)
Numerical solutions of nonlinear Equations: Fixed point iteration method, convergence and error estimation. Modified Newton-Raphson method, Muller's method, Inverse interpolation method, error estimations and convergence analysis. (6)
Roots of polynomial equations: Bairstow method, Graeffe's root squaring method and their convergences. (3)
Ordinary Differential Equations: Initial value problems–Picard's successive approximation method, error estimation. Single-step methods – Euler's method and Runge-Kutta method, error estimations and convergence analysis. Multi-step method – Milne's predictor-corrector method, Adams-Bashforth method, Adams-Moulton method, Error estimation, Convergence and stability analysis, Finite difference method, Shooting method, Introduction to finite element method. (8)
Partial Differential Equations by Finite difference method: Explicit and implicit methods, Consistency, Convergence and stability, Lax theorem, Heat equation: Explicit and Implicit Crank-Nicholson methods, Wave equation: Explicit finite difference method, stability analysis.(8)

References:

1. K. E. Atkinson, An Introduction to Numerical Analysis, 2nd Edition, Wiley-India.
2. S. D. Conte and C. de Boor, Elementary Numerical Analysis - An Algorithmic Approach, 3 rd Edition, McGraw-Hill, 1981.
3. R. L. Burden and J. D. Faires, Numerical Analysis, 7 th Edition, Thomson, 2001.
4. Froberg, C. E. – Introduction to Numerical Analysis.
5. Hildebrand, F.B. – Introduction to Numerical Analysis.
6. Ralston, A. and Rabinowits, P. – A First Course in Numerical Analysis.
7. Atkinson, K. and Cheney, W. – Numerical Analysis.
8. David, K. and Cheney, W. – Numerical Analysis.
9. Powell, M. – Approximation Theory and Methods.
10. Jain, M. F., Iyenger, S. R. K. and Jain, R.K. – Numerical Methods for Scientific and Engineering Computation.
11.Scheid, F. – Numerical Analysis.
12.Sanyal, D. C. and Das, K. - A Text Book of Numerical Analysis.
13. Reddy, J. N. – An Introduction to Finite Element Methods.
14.Sastry, S. S. – Introductory Methods of Numerical Analysis

Semester-II

Core Course-207: Abstract Algebra

Cayley's theorem. Conjugacy classes and class equation, p-groups. Converse of Lagrange's theorem for finite abelian groups. Sylow's theorems and its applications. Direct product, finitely generated abelian groups. Solvable groups – solvability of S_n, Jőrdan-Holder Theorem.
Ideals, Principal Ideal Domain (PID). Quotient ring, isomorphism and correspondence theorems. Prime, primary and maximal ideals – examples, characterizations and their interrelations. Prime and irreducible elements. Unique Factorization Domain (UFD). Ring with chain conditions – Noetherian rings and Artinian rings. Polynomial ring, Semi Simple Ring, Jacobson's radical, Hilbert basis theorem.
Field extension – algebraic and transcendental extension. Splitting field, algebraic closure and algebraically closed field . Separable and normal extension. Galois field.
Galois theory (If time permits) – introduction, basic ideas and results focusing the fundamental theorem of Galois theory. Solvability by radicals.

References :

1. D.S. Malik, John M. Mordeson and M.K. Sen, Fundamentals of Abstract Algebra, McGraw Hill, International Edition, 1997.
2. Dummit and Foote, Abstract Algebra, John Wiley and Sons, Inc.
3. T. H. Hungerford, Algebra , Springer Verlag
4. John B. Fraleigh , A first course in Abstract Algebra , Narosa.
5. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd, New Delhi, 1975
6. S. Lang, Abstract Algebra, 2nd edition, Addition -Wesley .
7. Joseph Gallian, Contemporary Abstract Algebra

Core Course-208: Functional Analysis

Metric spaces. Brief discussions of continuity, completeness, compactness. Hölder's and Minkowski's inequalities (statement only). Cantor's intersection theorem (statement only).
(3)
Baire's (category) theorem. The spaces R^n, C^n, l_p and $C[a,b]$. (3)
Contraction Mapping, Banach's fixed point theorem, applications to solutions of certain systems of linear algebraic equations, Fredholm's integral equation of the second kind, implicit function theorem. Kannan's fixed point theorem. (4)
Real and Complex normed linear spaces. Banach spaces, Riesz's lemma. Finite dimensional normed linear spaces, completeness, compactness criteria. Equivalent norms. (7)
Linear operators, Linear operators on normed linear spaces, continuity, bounded linear operators, norm of an operator, various expressions for the norm. Spaces of bounded linear operators. Inverse of an operator. (5)
Linear functionals. Hahn-Banach theorem, simple applications. Normed conjugate space and separability of the space. Uniform boundedness principle, simple application. The graph of an operator. Open mapping theorem and closed graph theorem. (6)
Inner product spaces, Cauchy Schwarz's inequality, the induced norm, polarization identity, parallelogram law. Orthogonality, Pythagoras Theorem, orthonormality, Bessel's inequality and its generalisation. (6)
Hilbert spaces, orthogonal complement, projection theorem. The Riesz's representation theorem and its converse. Convergence of series corresponding to orthogonal sequences, Fourier coefficient, Perseval's identity. (6)

References:

1. E. Kreyszig: Introductory Functional Analysis with Applications.
2. W. Rudin: Functional Analysis.
3. A. E. Taylor: Introduction to Functional Analysis.
4. B. K. Lahiri: Elements of Functional Analysis.
5. B. V. Limaye: Functional Analysis.
6. K. Yoshida: Functional Analysis.

Core Course-209: Topology

Quick Revision: Countable and uncountable sets. Axiom of choice and its equivalence, Cardinal numbers, Schroeder-Bernstein theorem, Continuum hypothesis. Zorn's lemma and well-ordering theorem, Ordinal Numbers, The first uncountable ordinal. (3)
Quick Revision: Metric spaces. Continuity, completeness, compactness. Cantor's intersection theorem (statement only).(3)
Topological spaces: Definition and examples of topological spaces, Open and Closed sets, Basis for a given topology, necessary andsufficient condition for two bases to be equivalent and sub-bases. Closure and Interior – their properties and relations; Exterior, Boundary, Accumulation points, Derived sets, Adherent point, Dense set, $G\delta$ and $F\sigma$ sets. Neighbourhoods and neighbourhood system, Subspace topology, Induced or Relative topology. (12)
Continuous, open, closed mappings, examples and counter examples, their differentcharacterizations and basic properties, Pasting lemma, homeomorphism, topologicalproperties. (8)
Connectedness: Examples, various characterizations and basic properties. Connectedness on the real line. Components and quasi components. Path connectedness and path components (8)
Compactness: Characterizations and basic properties of compactness, Lebesgue lemma. Sequential compactness, BW Compactness and countable compactness. Local compactness and Baire Category Theorem, Tychonoff theorem (on Product of Compact Spaces).(8)
The countability axioms, Separation axioms, Lindeloffness and their relationships. Ti spaces (i = 0, 1, 2, 3, 3½, 4, 5), their characterizations and basic properties.Normal spaces, Urysohn'S lemma and Tietzes extensiontheorem (Statements only) and some of their applications. (8)

<u>References :</u>

1) Basic Topology, M. A. Armstrong, Springer (India), 2004.
2) Topology, J.R. Munkres, 2nd Ed., PHI (India), 2002.
3) Introduction to Topology and Modern Analysis, G.F. Simmons, McGraw- Hill, New York.
4) General Topology by J. L. Kelley, Van Nostrand.
5) A text book of Topology by B. C. Chatterjee, S. Ganguly and M. Adhikari, Asian Books
6) Introduction to General topology by K. D. Joshi, Wiley Eastern Ltd.

Core Course-210: Partial Differential Equations

Origins of Partial Differential Equations (PDE). Linear and non- linear PDE. Cauchy's method of characteristics , Charpit's method, Jacobi's method.
Second order PDE with constant and variable coefficients. Reduction to canonical forms and

Classification, characteristic curves. Well-posed and ill-posed problems. Non linear PDE of second order.
Wave equation: vibrations of strings, D'Alembert's solution, Riemann's method, Solution by separation of variables, Transverse vibrations of membranes.
Laplace Equation: Equipotential surfaces, Boundary value problems, Maximum-minimum principles, The Cauchy problem, Stability of the solution. Theory of Green's function.
Diffusion equation: Boundary value problems, variables separable solution. Duhamel's Principle.
Solution of linear partial differential equations by Lie algebraic method.

References :

1. Sneddon I. N. : Elements of Partial Differential Equations, Mcgraw Hill.
2. Petrovsky I. G. : Lectures on Partial differential equations.
3. Courant and Hilbert : Methods of Mathematical Physics, Vol – II.
4. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society.
5. F. John, Partial Differential Equations, Narosa.
6. Williams W. E. : Partial Differential Equations.
7. Miller F. H. : Partial Differential Equations.
8. K.S. Rao, Introduction to partial differential equations, Prentice Hall, New Delhi, 1997.
9. Zafar Ahsan, Differential Equation and their applications, PHI Learning , New Delhi.

Core Course-211: Operation Researchs

Linear Programming: Fundamental theorem of L.P.P. along with the geometry in n-dimensional Euclidean space (hyperplane, separating and supporting plane).
Standard forms of revised simplex method, Computational procedure, Comparison of simplex method and revised simplex method, The Dual Simplex Method.
Mathematical formulation of Assignment Problem, Optimality condition, Hungarian method, Maximization case in Assignment problem, Unbalanced Assignment problem, Restriction on Assignment, Travelling salesman problem.(15L)
Sensitivity Analysis: Changes in price vector of objective function, changes in resource requirement vector, addition of decision variable, addition of a constraint. (5L)
Integer Programming (IP): The concept of cutting plane for linear integer programs, Gomory's cutting plane method, Gomory's All-Integer Programming Method, Branchand Bound Algorithm for general integer programs.(4)
Sequencing Models: The mathematical aspects of Job sequencing and processing problems, Processing n jobs through Two machines, processing jobs through machines. (4L)
Network Analysis: Network definitions, Minimal Spanning Tree Algorithm, Shortest Route Algorithms, A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis, PERT analysis in controlling project.(10L)
Queueing Theory: Basic features of queueing systems, operating characteristics of a queueing system, arrival and departure (birth & death) distributions, inter-arrival and service times distributions, transient, steady state conditions in queueing process. Poisson queueing models M/M/1, M/M/C for finite and infinite queue length. (12L)

References:

1. H. A. Taha - Operations Research-An Introduction. Macmillan Pub. Co., Inc., New York.
2. G. Hadley -– Linear Programming, Narosa Publishing House

3. S. S. Rao - Optimization Theory and Application, Wiley Eastern.
4. K Sarup, P. K. Gupta and Man Mohan - Operation Research, Sultan Chand & Sons.
5. J. K. Sharma - Operation Research, Mcmillan, India.
6. Schaum's Outline Series – Operations Research

SEC: Computer-aided Numerical practical Using C

Programming Problems :
1. Cubic spline interpolation.
2. Gauss Elimination Method for a System of Linear Equations.
3. Newton's Method for a System of Nonlinear Equations.
4. Inverse of a matrix.
5. Integration by Romberg's method.
6. Largest Eigen values of a real matrix by power Method.
7. Numerical Solutions of Ordinary Differential Equations for Initial Value Problems : (a) Picard's Formula, (b) Adams-Bashforth method, (c) Milne's predictor–corrector method.
8. Finite Difference Method for PDE – Elliptic Type PDE, Parabolic Type PDE, Hyperbolic Type PDE.

References :
1. Computing methods - Berzin and Zhidnov.
2. Analysis of Numerical methods - Isacson and Keller.
3. A first course in Numerical Analysis - Ralston and Rabinowitz.
4. Numerical solution of differential equations - M. K. Jain.
5. Numerical solution of partial differential equations- G. D. Smith.
6. Theory and Problems of Numerical Analysis - F. Scheid
7. Applied numerical methods using Matlab, W. Y. Yang, W. Cao, T-S Chun and J. Morris, Wiley-Interscience, John Wiley & Sons, 2005.

Semester-III

Core Course-312: Graph Theory, Integral Equation and Integral Transformation

Graph Theory
Definition of graphs and Digraphs, circuits, cycles, sub-graphs, induced sub-graphs. Degree of vertex. Connectivity, Components, Complete and complete bipartite graphs, Euler graph, Hamiltonian Graph and their properties. Trees their related theorems, Spanning tree, Rooted tree, Binary search tree, Tree traversal, Matrix representation of graphs, Fundamental cut set and cycles, Planar graphs and their properties, Euler's formula for connected graphs. (20)

Integral Equation
Symmetric, separable, iterated and resolvent kernel, Fredholm and Voltera integral equation & their classification, integral equation of convolution type, eigen value & eigen function, method of converting an initial value problem (IVP) into a Voltera integral equation, method of converting a boundary value problem (BVP) into a Fredholm integral equation, homogeneous Fredholm integral equation of the second kind with separable or degenerate kernel; classical Fredholm theory-Fredholm alternative, Fredholm theorem. (5)
Method of successive approximations: Solution of Fredholm and Voltera integral equation of the second kind by successive substitutions & Iterative method (Fredholm integral equation only),

reciprocal function, determination of resolvent kernel and solution of Fredholm integral equation. (2)
Hilbert-Schmidt theory: Orthonormal system of function, fundamental properties of eigen value and function for symmetric kernel, Hilbert theorem, Hilbert-Schmidt theorem. (2)

Integral Transform
Laplace transforms of elementary functions & their derivatives and Dirac-delta function, Laplace integral, Lerch's theorem (statement only), property of differentiation, integration and convolution, inverse transform, application to the solution of ordinary differential equation, integral equation and BVP. (5)
Fourier Transform: Fourier transform of some elementary functions and their derivatives, inverse Fourier transform, convolution theorem & Parseval's relation and their application, Fourier sine and cosine transform; Hankel Transform, inversion formula and Finite Hankel transform, solution of two-dimensional Laplace and one-dimensional diffusion & wave equation by integral transform. (6)

References:
1. J. P Tremblay and R. Manohar**:** Discrete Mathematical Structures with Applications to Computers.
2. C. L. Liu **:**Elements of Discrete Mathematics.
3. F. Harary**:** Graph Theory.
4. C. Berge **:**The theory of Graphs and its Applications.
5. N. Deo**:** Graph Theory with Applications to Engineering and Computer Science.
6. K . D. Joshi **:**Foundation of Discrete Mathematics.
7. L. S. Levy**:** Discrete Structure in computer Science.
8. D. G. West: Graph Theory.
9. M. D. Raisinghania: Integral Equations and Boundary Value Problems.
10. R. P. Kanwal**:** Linear Integral Equations.
11. S. G. Michelins**:** Linear Integral Equations.
12. D. V. Wider**:** The Laplace Transforms.
13. P. J. Collins: Differential and Integral Equations.
14. H. S. Carslaw and J. C. Jaeger**:** Operational Methods in Applied Mathematics.
15. I. G. Petrovsky **:** Lectures on the Theory of Integral Equations.
16. R. V. Churchill**:** Operational Mathematics.
17. L. Debnath and D. Bhatta : Integral Transforms and Their Applications.
18. I. N. Sneddon **:** The Use of Integral Transforms.
19. B. Davies: Integral Transforms and Their Applications.
20. A. M. Wazwaz : A First Course in Integral Equations.
21. N. V. Mclachlan **:** Operational Calculus.

Core Course-313: Continuum Mechanics

Theory of strain: Notion of continuum; Deformation – material and spatial methods; Measure of deformation – strain tensor, infinitesimal strain tensor, geometric interpretation; Strain vector – strain quadric, principal strains, strain invariants, strain quadric, Compatibility equations for linear strains. [10]
Theory of stress: Forces on a continuum; Stress tensor – stress quadric, principal stresses, stress invariants,equations of equilibrium. [7]

Motion of a continuum: Conservation of mass - equation of continuity; Principle of balance of linear momentum; principal of balance of angular momentum; conservation of energy, First law of Thermodynamics. [6]

Theory of elasticity: Ideal materials, classical elasticity, Generalized Hooke's Law, Isotropic materials, Constitutive equation (stress-strain relations) for isotropic elastic solid, Strain-energy function, Beltrami-Michel compatibility equations for stresses, field equations of linear elasticity, Fundamental boundary value problems of elasticity and uniqueness of their solutions (Statement only), Saint-Venant's principle. [10]

Fluid media: Path lines, stream lines and streak lines, Bounding surface, Lagrange's criterion for bounding surface. [7]

Water waves: General features, Condition at the free surface, Cisotti's equation, Complex potential, energy and path of particles for progressive waves and stationary waves, group velocity. [10]

Reference:

1. Mechanics of Continua. A. C. Eringen (Wiley, 1967).
2. Mathematical Theory of Continuum Mechanics. R. N. Chatterjee (Narosa Publishing House, New Delhi, 1999).
3. Mathematical theory of Elasticity. I. S. Sokolnikoff, (Tata Mc Grow Hill Co., 1977).
4. Continuum Mechanics. D. S. Chandrasekharaiah and L. Debnath (Academic Press, 1994).

Core Course-314: Calculus of $\mathbb{R}^n$

Differentiation on $\mathbb{R}^n$: Directional derivatives and continuity, the total derivative and continuity, total derivative in terms of partial derivatives, the matrix transformation of T : $\mathbb{R}^n \to \mathbb{R}^n$. The Jacobian matrix. (10)

The chain rule and its matrix form. Mean value theorem for vector valued function. Mean value inequality. (5)

A sufficient condition for differentiability. A sufficient condition for mixed partial derivatives. (5)

Functions with non-zero Jacobian determinant, the inverse function theorem, the implicit function theorem as an application of Inverse function theorem. Extremum problems with side conditions – Lagrange's necessary conditions as an application of Inverse function theorem. (10)

Integration on $\mathbb{R}^n$: Integral of f : A→ R when A$\subset\mathbb{R}^n$ is a closed rectangle. Conditions of inerrability. Integrals of f : C →R, C $\subset\mathbb{R}^n$ is not a rectangle, concept of Jordan measurability of a set in $\mathbb{R}^n$.

Fubini's theorem for integral of f : A × B , A$\subset\mathbb{R}^n$, B$\subset\mathbb{R}^n$ are closed rectangles.

Fubini's theorem for f : C →R, C ⊂ A × B.

Formula for change of variables in an integral in $\mathbb{R}^n$. (10)

References :

1. T. M. Apostol :Mathematical Analysis.
2. M. Spivak:Calculus on Manifolds.
3. W. Rudin: Principles of Mathematical Analysis

Core Course-315: Differential Manifold (P)

Differentiable manifolds: basic notions; the effects of second countability and Hausdorffness; tangent and cotangent spaces; submanifolds; consequences of the Inverse Function Theorem; vector fields and their flows; the Frobenius Theorem; Sard's theorem.

Differential forms: Multilinear algebra; tensors; differential forms; the de Rham complex and its behaviour under differentiable maps; the Lie derivative; differential ideals.

Lie groups: Lie groups; Lie algebras; homomorphisms; Lie subgroups; coverings of Lie groups; the exponential map; closed subgroups; the adjoint representation; homogeneous manifolds. Integration on manifolds: orientation; the integral of differential forms on differentiable singular chains; integration of differential forms of top degree on an oriented 3 differentiable manifold; the theorems of Stokes; the volume form on an oriented Riemannian manifold; the divergence theorem; integration on a Lie group. de Rham cohomology: definition; real differentiable singular cohomology; statement of the de Rham theorem; the Poincar lemma.

References :

1. M. Spivak; A Comprehensive Introduction to Differential Geometry, Vols I-V; Publish or Perish, Inc. Boston, 1979
2. J.A. Thorpe Elementary topics in Differential Geometry, Under - graduate Texts in Mathematics, Springer
Verlag, 1979.
3. Kobayashi. S. and Nomizu. K. Foundations of Differential Geometry, Interscience Publishers, 1963
4. F. W. Warner, Foundations of differentiable manifolds and Lie groups.
5. Christian Br; Elementary Differential Geometry; Cambridge University Press, 2011.
6. I. Madsen and J. Tornehave, From calculus to cohomology, Cambridge University Press.

Core Course-315: Dynamical System (A)

Autonomous and non-autonomous systems: Orbit of a map, fixed point, equilibrium point, periodic point, circular map, configuration space and phase space. (8)
Nonlinear oscillators-conservative system. Hamiltonian system. Various types of oscillators in nonlinear system viz. simple pendulum, and rotating pendulum. (5)

Limit cycles: Poincaré-Bendixon theorem (statement only). Criterion for the existence of limit cycle for Liénard's equation. (4)
Stability: Definition in Liapunov sense. Routh-Hurwitz criterion for nonlinear systems. Liapunov's criterion for stability. Stability of periodic solutions. Floquet's theorem. (10)
Solutions of nonlinear differential equations by perturbation method: Secular term. Nonlinear damping. Solutions for the equations of motion of a simple pendulum, Duffing and Vanderpol oscillators. (5)

Bifurcation Theory: Origin of Bifurcation, Bifurcation Value, Normalisation, Resonance, Stability of a fixed point. Bifurcation of equilibrium solutions – the saddle node bifurcation, the pitch-fork bifurcation, Hopf-bifurcation. (5)

Randomness of orbits of a dynamical system: The Lorentz equations, Chaos, Strange attractors. (3)

References:
1. D. W. Jordan and P. Smith: Nonlinear Ordinary Differential Equations.
2. F. Verhulst: Nonlinear Differential Equations and Dynamic Systems.
3. R. L. Davaney: An Introduction to Chaotic Dynamical Systems.
4. P. G. Drazin: Non-linear Systems.
5. K. Arrowsmith: Introduction to Dynamical Systems.
6. C. Havyshi: Nonlinear Oscillations in Physical Systems.
7. A. H. Nayfeh and D. T. Mook: Nonlinear Oscillations.
8. V. I. Arnold: Dynamical Systems V-Bifurcation Theory and Catastrophy Theory.
9. V. I. Arnold: Dynamical Systems III – Mathematical Aspects of Classical and Celestial Mechanics.

Core Course-316: Operator Theory (P)

Conjugate Space: Definition of conjugate space, determination of conjugate spaces of R^n, C^n, l_p for $1 \leq p < \infty$. Representation theorem for bounded linear functionals on $C[a,b]$ (Statement only). Some idea about the spaces $B[a,b], BV[a,b]$. Determination of conjugate space of $C[a,b]$ and some other finite and infinite dimensional spaces. (10)

Weak convergence and weak* convergence: Definition, characterization of weak convergence and weak* convergence, sufficient condition for the equivalence of weak* convergence and weak convergence in the dual space. Some idea of weak Cauchy sequence and weak convergence. (6)

Reflexive spaces: Definition of reflexive space, canonical mapping, relation between reflexivity and separability, some consequences of reflexivity. Definition of Strictly convex Banach space and Uniformly convex Banach space and their relations. (6)

Bounded linear operator: uniqueness theorem, adjoint of an operator and its properties.
Self-adjoint operator, compact, normal, unitary and positive operators, norm of self adjoint operator, group of unitary operator, square root of positive operator-characterization and basic properties, projection operator and their sum , product & permutability, invariant subspaces. Closed linear transformation (10)

Unbounded operator: Basic properties, Cayley transform, change of measure principle, spectral theorem. (4)

Compact map: Basic properties, compact symmetric operator, Rayleigh principle, Fisher's principle, Courant's principle, Mercer's theorem, positive compact operator. (4)

Reference:
1. A. E. Taylor: Introduction to Functional Analysis.
2. E. Kreyszing: Introductory Functional Analysis with Applications.
3. B. V. Limaye: Functional Analysis.
4. P. K. Jain and O.P. Ahuja: Functional Analysis.
5. C. Bachman and L. Narici: Functional Analysis.
6. B. K. Lahiri: Elements of Functional Analysis.
7. W. Rudin: Functional Analysis. .
8. G. F. Simons: Introduction to Topology and Analysis.

Core Course-316: Theory of Electromagnetic Fields (A)

Electrostatic field $\vec{E}$ **:**Coulomb law, principle of superposition, divergence and curl of $\vec{E}$, boundary conditions; Electrostatic potential – Poission equation, energy in electrostatic field, electric dipole, conductor and insulator; $\vec{E}$ in dielectric media – electric polarization, divergence of displacement vector, energy in dielectric media. [10]

Magnetostatic field $\vec{B}$: Electric current, equation of continuity, Ohm's law, Lorentz force law, Biot-Savart law, divergence and curl of $\vec{B}$, boundary conditions; Magnetic vector potential – multi-pole expansion, magnetic dipole; $\vec{B}$ in matter – magnetization, auxiliary field $\vec{H}$, curl of $\vec{H}$. [8]

Electromagnetic induction: Faraday's law; inductance; energy in magnetic field. [4]

Maxwell's equations: Electrodynamics before Maxwell – Ampere-Maxwell equation; Maxwell's equations - in vacuum, in matter, physical significance, boundary conditions; Energy transfer and Poynting theorem. [10]

Electromagnetic waves: Wave motion – general features, phase velocity, group velocity; Plane wave solution of Maxwell's equations - electromagnetic waves in vacuum; Reflection and transmission of plane electromagnetic waves at the boundary between two linear media. [8]

General solution of Maxwell's equations: Electromagnetic potentials – gauging of potentials, representation of fields in terms of potentials; Retarded potentials; Jefimenko's equations; Dipole radiation; Radiation by point charges. [10]

Reference:

1. Griffiths D. J., Introduction to electrodynamics (3rd Edition), *PHI Learning Private Limited, New Delhi* (2012).
2. Holliday David, and Resnick Robert Physics, Part II, Welly Eastern Limited and New Age International Limited, New Delhi, (1984).
3. Coulson A. A., Electricity, *Oliver and Boyd, Edinberg & London* (1974).
Jackson H. D., Classical Electrodynamics, *John Wiley and Sons, Inc.*, New York (1962).

GEC: Statistical Techniques

Collection and Presentation of Data: Primary and Secondary data, Primary methods of data collection, Drafting Questions and Questionnaires, Source of Secondary Data. General rules for constructing diagrams, one dimensional diagram, two dimensional diagrams, Pictograms and Cartograms.

Measure of Central Tendency: Arithmetic mean, Median, Mode, Geometric mean and Harmonic mean, Merits and demerits.

Measure of Variation: Introduction, mean deviation, Standard deviation, merits and limitations.

Correlation Analysis: Types of Correlation, methods of studying correlation, Karl Person's co-efficient of correlation, Rank correlation co-efficient, methods of least squares.

Regression Analysis: Introduction, Regression lines, Regression equations of Y on X, Regression Co-efficient.

Probability: Random experiment, Sample space and events, Axioms of probability, Conditional probability and independence, Addition, Multiplication and Baye's theorem.

References:

1. Business Statistics : S.P. Gupta and M.P. Gupta
2. An Introduction to Statistical methods : C.B. Gupta

Semester-IV

DSE-417 Elective Course-1 (Pure Stream)

Advanced Functional Analysis-I

Hilbert Space: Preliminary concept of Pre-Hilbert space and Hilbert space. Bessel's inequality. Complete orthonormal sequence and separability in Hilbert spaces. Cardinality. Isometric isomorphism of every infinite dimensional separable Hilbert space with the space l_2. Grahm-Schimdt orthonormalization process. Stone-Weierstrass theorem. Approximation in normed linear spaces. Best approximation and its uniqueness. (15)
Conjugate Space: Preliminary ideas of conjugate space, isometrically isomorphic spaces. Determination of conjugate spaces of C, C_0 and $C[a,b]$. Representation theorem for bounded linear functional on $C[a,b]$. (10)
Reflexive Space: Definition of reflexive space. Canonical mapping or canonical embedding. Subspaces of reflexive space, Bounded sequence contains a weakly convergent subsequence. Existence of an element of smallest norm. Relation between separable spaces and reflexive spaces. Reflexivity of Hilbert spaces. Strictly convex Banach spaces and uniformly convex Banach spaces. Helley's theorem (statement only). Milman and Pettis theorem for uniformly convex Banach space (statement only). (15)

Reference:

1. G. Bachman and L. Narici **:** Functional Analysis.
2. A. L. Brown and A. Page **:** Elements of Functional Analysis.
3. J. B. Conway **:** A Course in Functional Analysis
4. E. Kreyszig **:** Introductory Functional Analysis with Applications
5. B. V. Limaye **:** Functional Analysis
6. W. Rudin **:** Functional Analysis.
7. B. K. Lahiri **:** Elements of Functional Analysis

Advanced Differential Geometry-I

Riemannian manifolds: Affine connections, torsion tensor and curvature tensor of an affine connection, Riemannian metrics, Riemannian manifold, fundamental theorem of Riemannian geometry, Riemannian connection, Bianchi identities, generalized and proper generalized curvature tensors, Ricci tensor, scalar curvature, Gaussian curvature, sectional curvature, Schur's theorem, isometry groups of Riemannian manifold, model spaces of Riemannian geometry, Einstein manifolds, quasi-Einstein manifolds and their generalizations. [20H]

Transformations on Riemann manifolds: Geodesics on Riemannian manifolds, Hopf-Rinow theorem, conformal transformations, projective transformations, concircular transformations, conharmonic transformations and their properties, semi-symmetric and quarter symmetric metric connections. [15H]

Theory of submanifolds: Submanifolds and hypersurfaces of Riemannian manifolds, induced connection and second fundamental form, Gauss and Weingarten formulae, equations of Gauss, Codazzi and Ricci, mean curvature, totally geodesic and totally umbilical submanifolds, minimal submanifolds. [15H]

Reference:

1. Riemannian Geometry. P. Petersen (Springer-Verlag, 2016).
2. Geometry of Submanifolds. B. Y. Chen (Dover Publications Inc., 2019).
3. Riemannian Manifolds, An Introduction to Curvature. J. M. Lee (Springer-Verlag, 2005).
4. Differential Geometry of Manifolds. U. C. De & A. A. Shaikh (Narosa Publ. Pvt. Ltd, New Delhi, 2007).
5. Foundations of Differential Geometry, Vol. 2. S. Kobayashi & K. Nomizu (Interscience Press, New York, 1969).
6. Riemannian Geometry. T. J. Willmore (Oxford University Press, 1997).
7. Structure on Manifolds, K. Yano & M. Kon (World Scientific Publication, Singapore, 1984).
8. Riemannian Geometry. Manfredo P. Do Carmo (Birkhauser, Boston, 1992).

DSE-417 Elective Course-1 (Applied Stream)

Advanced Operations Research-I

Network Analysis: Network definitions, Minimal Spanning Tree Algorithm, Shortest Route Algorithms, Max-flow Min-cut theorem, Generalized Max-flow Min-cut theorem, linear programming interpretation of Max-flow Min-cut theorem, minimum cost flows. A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis, PERT analysis in controlling project. (20)

Queuing Theory: Basic features of Queuing Systems, Operating characteristics of a Queuing System, Arrival and Departure (birth and death) distributions, Inter-arrival and service times distributions, Transient steady-state conditions in queuing process.

Poisson queuing models : (M / M / 1) : (/ FIFO /) ; (M / M / 1) : (N / FIFO /) ; (M / M / C) : (/ FIFO /) ; (M / M / C) : (N / FIFO /), C N ; (M / M / R) : (K / GD / K), R < K– machine servicing model; (12)

Simulation: A brief introduction to simulation, Advantages of simulations over traditional search methods, Limitations of simulation techniques, Computational aspects of simulating a system, random number generation in stochastic simulation, Monte-Carlo simulation and modeling aspects of a system, Simulation approaches to inventory and queuing systems.
(8)

Reference:

1. Operations Research – K. Swarup, P. K. Gupta and Man Mohan.
2. Operations Research – H. A. Taha.
3. Operations Research – S. D. Sharma.
4. Introduction to Operations Research – A. Frederick, F. S. Hillier and G. J. Lieberman.
5. Optimization Theory and Applications – S. S. Rao.
6. Engineering Optimization : Theory and Practice – S. S. Rao
7. Optimization Methods in Operation Research – K. V. Mital.
8. Queuing Theory – J. A. Panico.
9. Introduction to Theory of Queues – L. Takacs.

Mathematical Biology-I

Mathematical models in ecology: Discrete and Continuous population models for single species. Logistic models and their stability analysis. Stochastic birth and death processes. (6)

Continuous models for two interacting populations: Lotka-Volterra model of predator -prey system, Kolmogorov model. Trophic function. Gauss's Model. (5)
Leslie-Gower predator-prey model. Analysis of predator-prey model with limit cycle behavior, parameter domains of stability. Nonlinear oscillations in predator-prey system. (5)
Deterministic Epidemic Models: Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic- Kermack-Mckendrick Threshold Theorem. (6)
Delay Models: Discrete and Distributed delay models. Stability of population steady states. (5)
Spatial Models: Formulating spatially structured models. Spatial steady states: Linear and nonlinear problems. Models of spread of population. (6)
Blood flow models: Basic concepts of blood flow and its special characteristics. Application of Poiseulle's law to the study of bifurcation in an artery. (5)
Pulsatile flow of blood in rigid and elastic tubes. Aortic diastolic-systolic pressure waveforms. Moen-Korteweg expression for pulse wave velocity in elastic tube. Blood flow through artery with mild stenosis. (6)
Models in Pharmacokinetics: Compartments, Basic equations, single and two compartment models. (6)

References:

1. K. E. Watt: Ecology and Resource Management-A Quantitative Approach.
2. R. M. May: Stability and Complexity in Model Ecosystem.
3. A. Segel: Modelling Dynamic Phenomena in Molecular Biology.
4. J. D. Murray: Mathematical Biology. Springer and Verlag.
5. L. Perko (1991): Differential Equations and Dynamical Systems, Springer Verlag.
6. F. Verhulust (1996): Nonlinear Differential Equations and
Dynamical Systems, Springer Verlag.
7. H. I. Freedman - Deterministic Mathematical Models in Population Ecology.
8. Mark Kot (2001): Elements of Mathematical Ecology, Cambridge Univ. Press
9. Fung, Y.C.: Biomechanics.

DSE-418 Elective Course-2 (Pure Stream)

Advanced Complex Analysis-I

Periodic Function: Simply and doubly periodic functions. Limit point of period points. Fundamental periods. Congruent points. Geometrical representation of the periodicity of a doubly periodic function. Liouville's First, Second and Third theorems. Relation between zeros and poles of an elliptic function. Weierstrass Elliptic function and its properties. Differential equation satisfied by Weierstrass elliptic function. Addition theorem and its applications. (22)
Functions of Several Complex Variables: Power series in several complex variables. Region of convergence of power series. Associated radii of convergence. Analytic functions. Cauchy-Riemann equations. Cauchy's integral formula. Taylor's expansion. Cauchy's inequalities. Zeros and Singularities of analytic functions. (18)

Reference:

1. E. C. Tittmarsh, The Theory of Functions.
2. A. I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III).
3. W. Kaplan, An Introduction to Analytic Functions.
4. H. Cartan, Theory of Analytic Functions.
5. R. C. Gunning and H. Rossi, Analytic Functions of Several Complex Variables.

6. B. A. Fuks. An Introduction to the Theory of Analytic Functions of Several Complex Variables.
7. Bochner and Martin. Several Complex Variables.

Advanced Algebra-I

Modules Theory:

Modules and Module Homomorphisms, Submodules and Quotient Modules, Operations on submodules, Direct Sum and Product, Finitely Generated Modules, Free Modules. Tensor Products of modules, Universal Property of the tensor product, Restriction and Extension of Scalars, Algebras. [10]
Exact Sequences, Projective, Injective and Flat Modules, Five Lemma, Projective Modules and $Hom_R(M,-)$, injective modules and $Hom_R(-,M)$, Flat modules and $M \otimes_R -$. [10]

Commutative Ring Theory:

Rings and Ring Homomorphisms, Ideals, Quotient Rings, Zero-divisors, Nilpotent elements, Units, Prime and Maximal ideals, Nilradical and Jacobson radical, Nakayama's Lemma, Operations on Ideals, Prime Avoidance, Chinese Remainder Theorem, Extension and Contraction of ideals. Rings and Modules of Fractions, Local Properties, Extended and contracted ideals in rings of fractions. Noetherian Rings, Primary Decomposition in Noetherian Rings. Integral Dependence, Lying-Over Theorem, Going-Up Theorem, Integrally Closed Domains, Going-Down Theorem, Noether Normalization, Hilbert Nullstellensatz. Transcendence Base, Separably Generated Extensions, Schmidt and Lüroth Theorems. [20]

References:

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.
5. Hungerford, T.W., Algebra, Springer.
6. Jacobson, N., Basic Algebra, II, Hindusthan Publishing Corporation, India.
7. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.
8. Curtis, C.W., Reiner, I., Representation Theory of Finite Groups and Associated Algebras, Wiley-Interscience, NY.

DSE-418 Elective Course-2 (Applied Stream)

Fluid Mechanics-I

Irrotational motion of fluid: Irrotational motion in simply connected and multiply connected regions, Kelvin's circulation theorem, Bernoulli's equations, Acyclic irrotational motion and some properties (Using Green's theorem). [6]

Motion in two-dimension: Stream function, Complex potential, Source, sink and doublets, Complex potentials for simple source, sink and doublet, Circle theorem, Uniform flow past a circle, Image of a source with respect to a plane boundary, image of a source outside a circle, image of a doublet outside a circle. [10]

Motion in three-dimension: Source, sink, doublet in three dimensions, motion of translation and rotation of circular cylinder in an infinite liquid, Blasius theorem, Kutta-Joukowski's theorem, axi-symmetric motion, Stokes' stream function, Three-dimensional motion. [9]

Vortex motion: Permanence of vortex lines and filaments, Equation of surface formed by the streamlines and vortex lines in the case of steady motion, Helmholtz's theorems, System of vortices, rectilinear vortices, Vortex pair and doublets, Image of vortex with respect to a circle, A single infinite row of vortices, Karman's vortex sheet, Pair of stationary vortex filament behind a circular cylinder in a uniform flow. [9]

Motion of viscous incompressible fluid: Viscous incompressible fluid flow: Field equations (Navier-Stokes' Equations), Boundary conditions, Reynolds number, Poisueile flow, Couette flow, Flow through parallel plates, Flow through pipes of circular and elliptic cross sections,Vorticity transport equation, Energy dissipation due to viscosity.[8]

Waves: Surface condition of gravity waves, Cisotti's equation, Complex potential, Small height gravity waves, Progressive waves- Cases of deep and shallow water, Stationary waves-possible wave lengths in a rectangular tank, Paths of particles for different waves, Energy for different waves. [8]

Reference:

1. G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 2005.
2. P. K. Kundu and I. M. Cohen, Fluid Mechanics, 4th Edition, Academic Press, 2008.
3. S.W. Yuan, Foundations of Fluid Mechanics, Prentice – Hall International, 1970.
4. J. L. Bansal, Viscous Fluid Dynamics, Oxford and IBH Publishing Co., 1977.
5. I. S. Sokolnikoff ,Mathematical theory of Elasticity, Tata Mc Grow Hill Co., 1977.

Theory of Relativity

Postulates and transformation of special relativity: Postulates of special theory of relativity; Lorentz transformations, general Lorentz transformations, mathematical properties of Lorentz transformations – length contraction, time dilation, simultaneity, twin paradox, Terrel effects, intervals, spacetime diagram, light cone, geometric representation of simultaneity [15]

Relativistic velocity and acceleration: Addition of velocity, transformation of velocity and acceleration, The Fizeau Effect, aberration of light, Doppler effect, relativistic mass and energy, Lorentz group–boosts, four-dimensional space-time: Proper time, four velocity, Lorentz transformation in four vector form. [10]

Relativistic Dynamics: Four force, four momentum, kinetic energy, mass-energy relation, forces in special theory of relativity, covariant formulation of Newton's law, transformation of momentum, photon, Compton effect. [10]

Relativistic electrodynamics: Transformations of electric and magnetic fields, equation of continuity,four current vector, invariance of Maxwell's equations; relativistic Lagrangian and Hamiltonian. [8]

Introduction to key concepts of general relativity: Principle of equivalence, idea of stress energy tensor, Ricci curvature, Einstein field equation: derivation from action, Schwarzchild metric: derivation. [7]

Reference:

1. Robert Resnick, Introduction to Special Relativity. 3rd edition (John Willey and Sons, 1968)
2. French, A. P. *Special Relativity*. Massachusetts Institute of Technology Education Research Center: MIT Introductory Physics Series. (New York, NY: Norton, 1968).
3. Taylor, Edwin F., and John A. Wheeler. *Exploring Black Holes: Introduction to General Relativity.*(San Francisco, CA: Addison Wesley Longman, 2000).
4. Wolfgang Rindler, Introduction to Special Relativity, 2nd ed,Oxford University Press.
5. Edwin F. Taylor and John Archibald Wheeler, Spacetime Physics: Introduction to Special Relativity, W. H. Freeman & Company, 1992
6. Einstein, Albert. *Relativity:The Special and the General Theory.* Translated by Robert W. Lawson. (New York, NY: Three Rivers Press/Random House, 1995).
7. Parthasarthy R. Introduction to general relativity. (Alpha Science International, Limited, 2016).

DSE-419 Elective Course-3 (Pure Stream)

Advanced Functional Analysis-II

Spectral Theory of Operators: Spectrum of a bounded linear operator. Resolvent set and Resolvent operator. Spectral radius. Spectral mapping theorem for polynomials. Spectrum of completely continuous operator and of self-adjoint operator. Spectral representation of self-adjoint operator. (15)

Banach Algebra: Banach algebra with identity. Examples. Resolvent operator and Resolvent function. Spectrum in Banach algebra. Topological divisor of zeros. Gelfand-Mazur theorem. Spectral mapping theorem. Complex homomorphism. Concept of Ideal in Banach algebra. (15)

Derivative of an Operator: Concept of Gateaux derivative and its uniqueness. Concept of Frechet derivative and its uniqueness. Representation of Gateaux derivative in case of finite domain and finite range. Relation between Gateaux derivative and Frechet derivative. Complete continuity of Frechet derivative. (10)

Reference:

1. G. Bachman and L. Narici **:** Functional Analysis.
2. A. L. Brown and A. Page **:** Elements of Functional Analysis.
3. J. B. Conway **:** A Course in Functional Analysis
4. E. Kreyszig **:** Introductory Functional Analysis with Applications
5. B. V. Limaye **:** Functional Analysis
6. W. Rudin **:** Functional Analysis.
7. B. K. Lahiri **:** Elements of Functional Analysis.

8. E. Rickart : Banach Algebra

Advanced Differential Geometry-II

Structures on Manifolds: Cartan's symmetric manifolds, recurrent manifolds, semi-symmetric manifolds, pseudosymmetric manifolds, Einstein field equation, Schwarchild spacetimes, Robertson-Walker spacetimes. [15H]
Complex Structures:Almost complex manifolds, Nijenhuis tensor, contravariant and covariant almost analytic vector fields, almost Hermite manifolds, Kähler manifolds, almost Tachibana manifolds, holomorphic sectional curvature, submanifolds of complex manifolds. [15H]
Contact Structures:Contact manifolds, K-contact manifolds, Sasakian manifolds, Kenmotsu manifolds, Trans-Sasakian manifolds, cosymplectic manifolds and their submanifolds. [20H]

Reference:

1. Contact Manifolds in Riemannian Geometry. D. E. Blair (Birkhausher, 2005).
2. Complex and Contact manifolds. U. C. De & A. A. Shaikh (Narosa Publ. Pvt. Ltd, New Delhi, 2009).
3. Geometry of Submanifolds. B. Y. Chen (Dover Publications Inc., 2019).
4. Structure on Manifolds. K. Yano & M. Kon (World Scientific, 1984).
5. Semi-Riemannian Geometry with Application to Relativity. B. O'Neill (Academic Press, 1983).
6. An introduction to contact topology. H. Geiges (Cambridge Univ. Press, 2008).
7. Contact geometry and non-linear differential equations. A. Kushner, V. Lychagin & V. Rubtsov (Cambridge Univ. Press, 2007).

DSE-419 Elective Course-3 (Applied Stream)

Advanced Operations Research-II

Markovian Decision Process: Markov chain, stochastic matrices, Power of stochastic matrices, regular matrices, Ergodic matrices, State transition diagram, imbedded Markov Chain method for Steady State solution. (6)

Reliability theory: Elements of Reliability theory, failure rate, extreme value distribution, analysis of stochastically falling equipments including the reliability function, reliability and growth model, MTTF, Linear increasing hazard rate, System reliability, Series configuration, Parallel configuration, Mixed configuration, Redundancy. (8)

Geometric Programming (GP): Posynomial, Signomial, Degree of difficulty, Unconstrained minimization problems, Solution using Differential Calculus, Solution seeking Arithmetic-Geometric inequality, Primal dual relationship & sufficiency conditions in the unconstrained case, Constrained minimization, Solution of a constrained Geometric Programming problem, Geometric programming with mixed inequality constrains, Complementary Geometric programming. (10)

Inventory Control: A brief introduction to Inventory Control, Single-item deterministic models without shortages and with shortages, inventory models with price breaks. Dynamic Demand Inventory Models. (10)

Single-item stochastic models without Set-up cost and with Set-up cost. (3)

Multi-item inventory models with the limitations on warehouse capacity, Average inventory capacity, Capital investment. (3)

References:

1. An Introduction to Information Theory – F. M. Reza.
2. Operations Research: An Introduction – P. K. Gupta and D.S. Hira.
3. Operations Research –K. Swarup, P. K. Gupta and Man Mohan.
4. Operations Research – K. Swarup, P. K. Gupta and Man Mohan.
5. Operations Research – H. A. Taha.
6. Operations Research – S. D. Sharma.
7. Introduction to Operations Research – A. Frederick, F. S. Hillier and G. J. Lieberman.
8. Optimization: Theory and Applications – S. S. Rao.

Mathematical Biology-II

Diffusion Model: The general balance law, Fick's law, diffusivity of motile bacteria. (5)
Models for Developmental Pattern Formation: Background, model formulation, spatially homogeneous and inhomogeneous solutions, Turing model, conditions for diffusive stability and instability, pattern generation with single species model. (5)
Effect of Nutrients on autotroph-herbivore interaction: Introduction, Models on nutrient recycling and its stability, Effect of nutrients on autotroph herbivore stability,
Models on herbivore nutrient recycling on autotrophic production. Models on phytoplankton-zooplankton system and its stability, Bio-control in plankton models with nutrient recycling. Leslie-Gower predator-prey model with different functional responses. (5)
Continuous models for three or more interacting populations: Food chain models.
Stability of food chains. Species harvesting in competitive environment, Economic aspects of harvesting in predator-prey systems. (5)
Interaction of Ratio-dependent models: Introduction, May's model, ratio-dependent models of two interacting species, two prey- one predator system with ratio-dependent predator influence- its stability and persistence. (5)
Microbial population model: Microbial growth in a chemostat. Stability of steady states.
Growth of microbial population. Product formation due to microbial action. Competition for a growth- rate limiting substrate in a chemostat. (5)
Deterministic Epidemic Models: Recurrent epidemics, Seasonal variation in infection rate, allowance of incubation period. Simple model for the spatial spread of an epidemic.
Proportional Mixing Rate in Epidemic: SIS model with proportional mixing rate, SIRS model with proportional mixing rate. Epidemic model with vaccination. (5)
Stochastic Epidemic Models: Introduction, stochastic simple epidemic model, Yule-Furry model (pure birth process), expectation and variance of infective, calculation of expectation by using moment generating function. (5)
Eco-Epidemiology: Predator-prey model in the presence of infection, viral infection on phytoplankton-zooplankton (prey-predator) system. (5)
Models for Population Genetics: Introduction, basic model for inheritance of genetic characteristic, Hardy-Wienberg law, models for genetic improvement, selection and mutation-steady state solution and stability theory. (5)

References:

1. J.D.Murray: Mathematical Biology, Springer and Verlag.
2. Mark Kot: Elements of Mathematical Ecology, Cambridge Univ. Press.
3. Leach Edelstein-Keshet: Mathematical Models in Biology, Birkhauser Mathematics Series.

4. F. Verhulust: Nonlinear Differential Equations and Dynamical Systems, Springer-Verlag.
5. R. M. May: Stability and Complexity in Model Ecosystem.
6. N.T.J.Bailey: The Mathematical Theory of Infectious Diseases and its Application, 2nd edn. London.
7. H. I. Freedman - Deterministic Mathematical Models in Population Ecology.
8. L.A.Segel (1984): Mpodelling Dynamical Phenomena in Molecular Biology, Cambridge University Press.
9. Vincenzo Capasso (1993): Lecture Notes in Mathematical Biology (Vol. No. 97)-Mathematical Structures of Epidemic Systems, Springer Verlag.
10. Eric Renshaw (1990): Modelling Biological Populations in Space and Time, Cambridge Univ. Press.
11. Busenberg and Cooke (1993): Vertically Transmitted Diseases- Models and Dynamics, Springer Verlag

DSE-420 Elective Course-4 (Pure Stream)

Advanced Complex Analysis-II

Harmonic functions: Definition of Harmonic function, Harmonic conjugate. Characterisation of Harmonic functions by mean-value property. Relation between harmonic function and analytic function. Construction of an analytic function. Poisson's integral formula. Poisson's kernel. Dirichlet problem for a disc. Harnack's inequality.(15)
Meromorphic Functions: Definition of Meromorphic functions and Entire functions. M(r, f) and its properties (statements only). Expansion of meromorphic function. Definition of the functions m (r, a), N(r, a) and T(r,f). Nevanlinna's first fundamental theorem. Cartan's identity and convexity theorems. Orders of growth. Order of a meromorphic function. Comparative growth of logM(r) and T(r). Nevanlinna's second fundamental theorem. Estimation of S(r) (Statement only). Deficient function. Nevanlinna's theorem on deficient functions. Nevanlinna's five-point uniqueness theorem. Milloux theorem. (25)

Reference:

1. E. C. Tittmarsh, The Theory of Functions.
2. A. I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III).
3. W. Kaplan, An Introduction to Analytic Functions.
4. H. Cartan, Theory of Analytic Functions.
5. W. K. Hayman, Meromorphic Functions.
6. L. Yang, Value Distribution Theory.
7. C. C. Yang and H. X. Yi, Uniqueness Theory of Meromorphic Functions.

Advanced Algebra-II

Multilinear Algebra:
Determinants, Tensor Algebras, Symmetric Algebras, Exterior Algebras, Homomorphisms of Tensor Algebras, Symmetric and Alternating Tensors. [6]
Structure of Rings:
Artinian rings, Simple rings, Primitive rings, Jacobson density theorem, Wedderburn - Artin theorem on simple (left)Artinian rings. [6]

The Jacobson radical, Jacobson semisimple rings, subdirect product of rings, Jacobson semisimple rings as subdirect products of primitive rings, Wedderburn - Artin theorem on Jacobson semisimple (left)Artinian rings. [10]
Simple and Semisimple modules, Semisimple rings, Equivalence of semisimple rings with Jacobson semisimple (left)Artinian rings, Properties of semisimple rings, Characterizations of semisimple rings in terms of modules. [10]

Group Representations:
Representations, Group-Rings, Maschke's Theorem, Character of a Representation, Regular Representations, Orthogonality Relations, Burnside Two-Prime Theorem. [8]

References:

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.
5. Hungerford, T.W., Algebra, Springer.
6. Jacobson, N., Basic Algebra, II, Hindusthan Publishing Corporation, India.
7. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.
8. Curtis, C.W., Reiner, I., Representation Theory of Finite Groups and Associated Algebras, Wiley-Interscience, NY.

DSE-420 Elective Course-4 (Applied Stream)

Fluid Mechanics-II

Basis electrodynamics: Electric field, Magnetic field, derivation of Maxwell's equations, work-energy theorem, electromagnetic potentials. [10]
Basic Equations: Derivation of basic equations of magnetohydrodynamics (MHD), MHD approximations, non-dimensional numbers, boundary conditions for velocity, temperature and magnetic field. [7]
Classical MHD: Alfven's theorem, Frozen-in-phenomenon-illustrative examples, Ferraro's law of isorotation. [6]
Magnetostatics: Force free magnetic field and related important results, illustrative examples on abnormality parameter-Chandrasekhar's theorem, Bennett pinch and instabilities associated with it. [8]
Alfven waves: Lorentz force as a sum of two surface forces- cause for Alfven waves, applications, Alfven wave equations in incompressible fluids, equipartition of energy, experiments on Alfven waves, dispersion relations, Alfven waves in compressible fluids, slow and fast waves, Hodographs. [12]
Flow Problems: Hartmann flow, Couette flow, Temperature distribution for these flows.[7]

Reference:

1. T. G. Cowling, Magnetohydrodynamics, Interscience, 1957.
2. V. C. A. Ferraro and C. Plumpton, An Introduction to Magneto-Fluid Mechanics, Oxford University Press, 1961.
3. G. W. Sutton and A. Sherman, Engineering Magnetohydrodynamics, McGraw Hill, 1965.

4. Alan Jeffrey, Magnetohydrodynamics, Oliver & Boyd, 1966.
5. K. R. Cramer and S. I. Pai, Magnetofluid Dynamics for Engineers and Applied Physicists, Scripta Publishing Company, 1973.
6. D. J. Griffiths, Introduction to Electrohydrodynamics, Prentice Hall, 1997.
7. P. H. Roberts, An Introduction to Magnetohydrodynamics, Longman, 1967.
8. H. K. Moffat, Magnetic field generation in electrically conducting fluids, Cambridge University Press, 1978

Quantum Mechanics

Fundamental ideas of quantum mechanics: Nature of the electromagnetic radiation; Wave-particle duality - double-slitexperiment, quantum unification of the two aspects of light, matter waves; Wave functions and Schrodinger equation; Quantum description of particle - wave packet, uncertainty relation. [6]

Mathematical formalism of quantum mechanics: Wave function space – bases, representation; State space – bases,representation; Observables – R and P observables; Postulates of quantum mechanics. [5]

Physical interpretation of the postulates: Statistical interpretation – expectation values, Ehrenfest theorem, uncertaintyprinciple; Physical implications of the Schrodinger equation - evolution of physical systems, superposition principle,conservation of probability, equation of continuity; Solution of the Schrodinger equation – time evolution operator, stationary state, time-independent Schrodinger equation; Equations of motion – Schrodinger picture, Heisenberg picture, interactionpicture. [7]

Theory of harmonic oscillator: Matrix formulation – creation and annihilation operators; Energy values; Matrix representationin |n> basis; Representation in the coordinate basis; Planck's law; Oscillator in higher dimensions. [5]

Symmetry and conservation laws: Symmetry transformations – basic concepts, examples; Translation in space; Translation oftime; Rotation in space; Space inversion; Time reversal.[5]

Angular momentum: Orbital angular momentum - eigen values and eigen functions of L^2 and L_z; Angular momentumoperators $\boldsymbol{J}$– commutation relations, eigen values and eigen functions; Representations of the angular momentum operators. [5]

Spin: Idea of spin – Bosons, Fermions; Spin one-half – eigen functions, Pauli matrices; Total Hilbert space for spin-halfparticles; Addition of angular momenta; Clebsch-Gordan coefficients – computaion, recursion relations, constructionprocedure; Identical particles - symmetrisation postulate, Pauli exclusion principle, normalization of states. [5]

One-electron atom: Schrodinger equation; Energy levels, Eigen functions and bound states, Expectation values and virialtheorem; Solution in parabolic coordinates; Special hydrogenic atom (brief description) – positronium, muonium, antihydrogen, Rydberg atoms. [4]

Relativistic quantum mechanics: Klein-Gordon equation – plane wave solution, interpretation of K-G equation; Dirac equation– covariant form, charged particle in electromagnetic field, equation of continuity, plane wave solution; Dirac hole theory; Spinof the Dirac particle.

[8]

Reference:

1. Quantum Mechanics Vol. 1. C. Cohen-Tannoudji, B. Diu, and F. Laloe, Wiley- Interscience publication, 1977.
2. Lectures on Quantum Mechanics. A. Das, Hindusthan Book Agency, New Delhi, 2003.
3. Quantum Mechanics.B. H. Bransden and C. J. Joachain, Prentics Hall (2005); Physics of Atoms and Molecules, PearsonEducation , 2007.
4. Introduction to Quantum Mechanics.D. J. Griffiths, Pearson Prentics Hall, Upper Saddle River, NJ, 2005.
5. Quantum Mechanics.L. I. Schiff, McGraw-Hill, New York, 1968.